

Rev 25 Taylor Series

centered on $x=a$:

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

Lagrange Error Bound

$$\frac{M(x-a)^{n+1}}{(n+1)!}$$

$$\frac{f^{(n)}(a)}{n!}(x-a)^n + \dots \text{ Remainder (error)}$$

$$M = \text{Max of } f^{(n+1)}(x)$$

 $a=0$: Maclaurin Series

Big 4

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} \dots \text{ ns40}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} \dots \text{ ns40}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n x^{2n}}{(2n)!} \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^{n+1} x^{n+1}}{(n+1)!} \dots \text{ ns41}$$

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make your own series GED $\frac{a}{1-r} = a + ar + ar^2 + \dots$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots + x^n + \dots$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots$$

Ex1. a) write the first 4 terms & general term for Taylor Series centered at $x=0$

$$f(x) = 6e^{-x/3}$$

b) $g(x) = \int_0^x f(t) dt$ write the Taylor Series for $g(x)$

$$a) \quad 6 \cdot 1 + 6\left(-\frac{x}{3}\right) + 6\left(\frac{-x}{3}\right)^2 \frac{1}{2!} + 6\left(\frac{-x}{3}\right)^3 \frac{1}{3!} + \dots + \frac{6\left(-\frac{x}{3}\right)^n}{n!}$$

$$6 - \frac{6x}{3} + \frac{6x^2}{3^2 \cdot 2!} - \frac{6x^3}{3^3 \cdot 3!} + \dots + \frac{(-1)^n x^n \cdot 6}{3^n \cdot n!}$$

$$b) \quad \int_0^x 6 - \frac{6t}{3} + \frac{6t^2}{3^2 \cdot 2!} - \dots dt$$

$$6t - \frac{6t^2}{3 \cdot 2} + \frac{6t^3}{3^2 \cdot 2! \cdot 3} - \dots + \frac{(-1)^n t^{n+1} \cdot 6}{3^n \cdot n! \cdot (n+1)}$$

$$6x - \frac{6x^2}{3 \cdot 2} + \frac{6x^3}{3^2 \cdot 2! \cdot 3} - \dots + \frac{(-1)^n x^{n+1} \cdot 6}{3^n \cdot n! \cdot (n+1)} - 0$$

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$$\text{Ex 2} \quad f(x) = 1 + (x+1) + (x+1)^2 + (x+1)^3 + \dots$$

a) find the sum of the series

b) $g(x) = \int_{-1}^x f(t) dt$. Find the series for $g(x)$

a) GEO $a=1$ $r=(x+1)$ $\text{sum} = \frac{1}{1-(x+1)} = -\frac{1}{x}$

b) $t + \frac{(t+1)^2}{2} + \frac{(t+1)^3}{3} \dots \Big|_{-1}^x$

$$\left[x + \frac{(x+1)^2}{2} + \frac{(x+1)^3}{3} \dots \right] - \left[-1 \right]$$

$$(x+1) + \frac{(x+1)^2}{2} + \frac{(x+1)^3}{3} \dots$$

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